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Synchrotron radiation from a charge moving along a helix around a dielectric cylinder

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Abstract
In this paper, we investigate the radiation emitted by a charged particle moving along a helical orbit around a dielectric cylinder immersed in a homogeneous medium. Formulae are derived for the electromagnetic potentials, electric and magnetic fields, and for the spectral-angular distribution of the radiation in the exterior medium. It is shown that under the Cherenkov condition for dielectric permittivity of the cylinder and the velocity of the particle image on the cylinder surface, strong narrow peaks appear in the angular distribution for the number of quanta radiated on a given harmonic. At these peaks the radiated energy exceeds the corresponding quantity for a homogeneous medium by several dozen times. Simple analytic estimates are given for the heights and widths of these peaks. The results of numerical calculations for the angular distribution of the radiated quanta are presented and they are compared with the corresponding quantities for the radiation from a charge moving along a helical trajectory inside a dielectric cylinder.

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1. Introduction

The extensive applications of synchrotron radiation in a wide variety of experiments and in many disciplines (see [1–6] and references therein), motivate the importance of investigations for various mechanisms of controlling the radiation parameters. In particular, it is of interest to consider the influence of a medium on the spectral and angular characteristics of the radiation. It is well known that the presence of medium can essentially change the characteristics of the electromagnetic processes and gives rise to new types of phenomena such as Cherenkov, transition and diffraction radiations. Moreover, the operation of a number of devices assigned to production of electromagnetic radiation is based on the interaction of high-energy particles with materials (see, for example, [7]). The synchrotron radiation from a charged particle circulating in a homogeneous medium was considered in [8], where it has been shown that the interference between the synchrotron and Cherenkov radiations leads to remarkable effects. New interesting features arise in the case of inhomogeneous media.

Interfaces of media are widely used to control the radiation flow emitted by various systems. Well-known examples of this kind are the Cherenkov radiation of a charge moving parallel to a plane interface of two media or flying parallel to the axis of a dielectric cylinder, transition radiation, Smith–Purcell radiation. In [9], the radiation from charged particles flying over a surface acoustic wave generated on a plane interface between two media is studied. In a series of papers initiated in [10–12], we have investigated the influence of spherical and cylindrical boundaries between two dielectrics on the characteristics of the synchrotron radiation. In particular, the investigation of radiation from a charge rotating about/inside a dielectric ball showed [13] that when the Cherenkov condition for the ball material and particle speed is satisfied, there appear high narrow peaks in the spectral distribution of the number of quanta emitted to outer space at some specific values of the ratio of ball-to-particle orbit radii. In the vicinity of these peaks the rotating particle may generate radiation field quanta exceeding in several dozens of times those generated by particle rotating in a continuous, infinite and transparent medium having the same real part of permittivity as the ball material. Similar problems with the cylindrical symmetry have been discussed in [12, 14]. It is shown that under similar Cherenkov condition for permittivity of cylinder and the speed of particle gyrating about/inside the cylinder, high narrow peaks are present in the spectral-angular distribution for the number of radiated quanta. In the vicinity of these peaks the radiated energy exceeds the corresponding value for the homogeneous medium case by several orders of magnitude.

In recent papers [15–18], we have investigated a more general problem of the radiation from a charged particle in a helical trajectory inside a dielectric cylinder. The corresponding problem for a charge moving in the vacuum has been widely discussed in the literature (see, for instance, [5, 6] and references given therein) and the influence of a homogeneous dispersive medium is considered in [19]. This type of electron motion is used in helical undulators for generating electromagnetic radiation in a narrow spectral interval at frequencies ranging from radio or millimeter waves to x-rays (see [5, 6, 20]).

The present paper is devoted to the investigation of the radiation from a charge moving along a helix around a dielectric cylinder immersed in a homogeneous medium. This paper is organized as follows. In the following section, by using the formulae for the Green function from [12], we derive expressions for the vector potential and electromagnetic fields in the region outside the cylinder. The angular-frequency distribution of the radiation intensity in the surrounding medium is considered in section 3. In section 4, we discuss the features of the radiation intensity and the results of the numerical evaluations are presented. Section 5 concludes the main results of this paper.

2. Electromagnetic fields in the exterior region

Consider a point charge q moving along the helical trajectory of radius ρ_0 outside a dielectric cylinder with radius ρ_1 and with permittivity ε_0. We will assume that this system is immersed in a homogeneous medium with dielectric permittivity ε_1 (magnetic permeability will be taken to be unit). This type of motion can be produced by a uniform constant magnetic field directed along the axis of a cylinder, by a circularly polarized plane wave, or by a spatially periodic transverse magnetic field of a constant absolute value and a direction that rotates as a function of the longitudinal coordinate. In helical undulators the last configuration is used. In the cylindrical coordinate system (ρ, ϕ, z) with the axis z directed along the cylinder axis, the components of the current density created by the charge are given by the formula

$$j_l = \frac{q}{\rho} v_l \delta(\rho - \rho_0)\delta(\phi - \omega_0 t)\delta(z - v_\parallel t), \tag{1}$$

where $\omega_0 = v_\perp/\rho_0$ is the angular velocity of the charge, $v_\parallel$ and $v_\perp$ are the particle velocities along the axis of the cylinder and in the perpendicular plane, respectively.

The solution to Maxwell equations for the vector potential is expressed in terms of the Green function $G_{il}(\mathbf{r}, t, \mathbf{r}', t')$ of the electromagnetic field by the formula

$$A_i(\mathbf{r}, t) = -\frac{1}{2\pi^2 c} \int G_{il}(\mathbf{r}, t, \mathbf{r}', t') j_l(\mathbf{r}', t')\, \mathrm{d}\mathbf{r}'\, \mathrm{d}t', \tag{2}$$

where the summation over l is understood. In accordance with the problem symmetry, for the Green function we have the following Fourier expansion:

$$\begin{aligned} G_{il}(\mathbf{r}, t, \mathbf{r}', t') = \sum_{m=-\infty}^{\infty} \int_{-\infty}^{\infty} \mathrm{d}k_z\, \mathrm{d}\omega G_{il}(m, k_z, \omega, \rho, \rho') \\ \times \exp[\mathrm{i}m(\phi - \phi') + \mathrm{i}k_z(z - z') - \mathrm{i}\omega(t - t')]. \end{aligned} \tag{3}$$

Substituting expressions (1) and (3) into formula (2), we present the vector potential as the Fourier expansion

$$A_l(\mathbf{r}, t) = \sum_{m=-\infty}^{\infty} \mathrm{e}^{\mathrm{i}m(\phi - \omega_0 t)} \int_{-\infty}^{\infty} \mathrm{d}k_z\, \mathrm{e}^{\mathrm{i}k_z(z - v_\parallel t)} A_{ml}(m, k_z, \rho), \tag{4}$$

where the coefficients $A_{ml} = A_{ml}(m, k_z, \rho)$ are given in terms of the Fourier components of the Green function by the formula

$$A_{ml} = -\frac{q}{\pi c}[v_\perp G_{l\phi}(m, k_z, \omega_m(k_z), \rho, \rho_0) + v_\parallel G_{lz}(m, k_z, \omega_m(k_z), \rho, \rho_0)]. \tag{5}$$

Here and in the discussion below we use the notation

$$\omega_m(k_z) = m\omega_0 + k_z v_\parallel. \tag{6}$$

By making use of the formula for the Green function given earlier in [12], in the Lorentz gauge for the corresponding Fourier components $G_{il} = G_{il}(m, k_z, \omega_m(k_z), \rho, \rho_0)$ in the region outside the cylinder, $\rho > \rho_1$, we find

$$\begin{aligned} G_{l\phi} &= \frac{\mathrm{i}^{2-\sigma_l}}{2} \sum_{p=\pm 1} p^{\sigma_l} B_m^{(p)} H_{m+p}(\lambda_1 \rho), \\ G_{lz} &= \frac{\mathrm{i}^{2-\sigma_l} k_z}{2\rho_1} \frac{H_m(\lambda_1 \rho_0) J_m(\lambda_0 \rho_1)}{\alpha_m V_m^H} \sum_{p=\pm 1} p^{\sigma_l - 1} \frac{J_{m+p}(\lambda_0 \rho_1)}{V_{m+p}^H} H_{m+p}(\lambda_1 \rho), \\ G_{zz} &= \frac{\pi}{2\mathrm{i} V_m^H} \left[J_m(\lambda_1 \rho_0) V_m^H - H_m(\lambda_1 \rho_0) V_m^J \right] H_m(\lambda_1 \rho), \end{aligned} \tag{7}$$

where in the first and second equations $l = \rho, \phi, \sigma_\rho = 1, \sigma_\phi = 2$,

$$\lambda_j^2 = \frac{\omega_m^2(k_z)}{c^2} \varepsilon_j - k_z^2, \qquad j = 0, 1, \tag{8}$$

$J_m(x)$ is the Bessel function, $H_m(x) = H_m^{(1)}(x)$ is the Hankel function of the first kind and

$$V_m^F = J_m(\lambda_0 \rho_1) \frac{\partial F_m(\lambda_1 \rho_1)}{\partial \rho_1} - F_m(\lambda_1 \rho_1) \frac{\partial J_m(\lambda_0 \rho_1)}{\partial \rho_1}, \qquad F = J, H. \tag{9}$$

In formulae (7) the coefficients $B_m^{(p)}$, $p = \pm 1$, are defined by the expressions

$$B_m^{(p)} = \frac{\pi}{2\mathrm{i}V_{m+p}^H}\left[J_{m+p}(\lambda_1\rho_0)V_{m+p}^H - H_{m+p}(\lambda_1\rho_0)V_{m+p}^J\right] + \frac{p\lambda_0 J_{m+p}(\lambda_0\rho_1)J_m(\lambda_0\rho_1)}{2\rho_1\alpha_m V_{m+p}^H}\sum_{l=\pm 1}\frac{H_{m+l}(\lambda_1\rho_0)}{V_{m+l}^H}, \tag{10}$$

with the notation

$$\alpha_m = \frac{\varepsilon_0}{\varepsilon_1 - \varepsilon_0} - \frac{\lambda_0 J_m(\lambda_0\rho_1)}{2}\sum_{l=\pm 1} l\frac{H_{m+l}(\lambda_1\rho_1)}{V_{m+l}^H}. \tag{11}$$

Note that the equation $\alpha_m = 0$ determines the eigenmodes of the dielectric cylinder.

In the definition of λ_1 given by (8), one should take into account that in the presence of the imaginary part $\varepsilon_1''(\omega)$ for the dielectric permittivity ($\varepsilon_1 = \varepsilon_1' + \mathrm{i}\varepsilon_1''$) the radiation field in the exterior medium should exponentially damp for large ρ. This leads to the following relations:

$$\lambda_1 = \begin{cases} (\omega_m/c)\sqrt{\varepsilon_1 - k_z^2c^2/\omega_m^2}, & \omega_m^2\varepsilon_1/c^2 > k_z^2, \\ \mathrm{i}\sqrt{k_z^2 - \omega_m^2\varepsilon_1/c^2}, & \omega_m^2\varepsilon_1/c^2 < k_z^2. \end{cases} \tag{12}$$

For $\lambda_1^2 > 0$ the sign of λ_1 may also be determined from the principle of radiation (different signs of ωt and $\lambda_1\rho$ in the expressions of the fields) for large ρ.

By taking into account formulae (7), from (5) we obtain the expressions for the Fourier components of the vector potential:

$$\begin{aligned} A_{ml} &= -\frac{q\mathrm{i}^{2-\sigma_l}}{2\pi}\sum_{p=\pm 1} p^{\sigma_l}C_m^{(p)}H_{m+p}(\lambda_1\rho), \qquad l = \rho, \phi, \\ A_{mz} &= -\frac{qv_\parallel}{2\mathrm{i}V_m^H}\left[J_m(\lambda_1\rho_0)V_m^H - H_m(\lambda_1\rho_0)V_m^J\right]H_m(\lambda_1\rho), \end{aligned} \tag{13}$$

with the coefficients

$$C_m^{(p)} = \frac{v_\perp}{c}B_m^{(p)} + pv_\parallel k_z\frac{J_m(\lambda_0\rho_1)J_{m+p}(\lambda_0\rho_1)H_m(\lambda_1\rho_0)}{c\rho_1\alpha_m V_m^H V_{m+p}^H}. \tag{14}$$

The Fourier expansions similar to (4) may also be written for the electric and magnetic fields. In the case of the magnetic field the corresponding Fourier coefficients have the form

$$\begin{aligned} H_{ml} &= \frac{\mathrm{i}^{2-\sigma_l}qk_z}{2\pi}\sum_{p=\pm 1} p^{\sigma_l - 1}D_m^{(p)}H_{m+p}(\lambda_1\rho), \qquad l = \rho, \phi, \\ H_{mz} &= -\frac{q\lambda_1}{2\pi}\sum_{p=\pm 1} pD_m^{(p)}H_m(\lambda_1\rho), \end{aligned} \tag{15}$$

with the notation

$$D_m^{(p)} = C_m^{(p)} - \frac{v_\parallel\lambda_1\pi}{2\mathrm{i}ck_zV_m^H}\left[J_m(\lambda_1\rho_0)V_m^H - H_m(\lambda_1\rho_0)V_m^J\right], \qquad p = \pm 1. \tag{16}$$

The corresponding Fourier coefficients for the electric field are obtained from the Maxwell equation $\nabla\times\mathbf{H} = -\mathrm{i}\omega\varepsilon_1\mathbf{E}/c$ with the result

$$\begin{aligned} E_{ml} &= \frac{\mathrm{i}^{1-\sigma_l}qc}{4\pi\omega_m(k_z)\varepsilon_1}\sum_{p=\pm 1} p^{\sigma_l}H_{m+p}(\lambda_1\rho)\left\{\left(\frac{\omega_m^2(k_z)\varepsilon_1}{c^2} + k_z^2\right)D_m^{(p)} - \lambda_1^2D_m^{(-p)}\right\}, \\ E_{mz} &= \frac{\mathrm{i}cq\lambda_1k_z}{2\pi\omega_m(k_z)\varepsilon_1}\sum_{p=\pm 1} D_m^{(p)}H_m(\lambda_1\rho), \end{aligned} \tag{17}$$

where, as before, $l = \rho, \phi$. Note that for purely transversal motion ($v_{\|} = 0$) one has $D_m^{(p)} = v_{\perp} B_m^{(p)}/c$. For a particle motion in a homogeneous medium we have $\varepsilon_0 = \varepsilon_1$ and the terms in the expressions of the coefficients $C_m^{(p)}$ and $B_m^{(p)}$ involving the function α_m vanish. In this case $V_m^J = 0$, $V_m^H = 2\mathrm{i}/\pi\rho_1$, and for the coefficients $D_m^{(p)}$ one finds

$$D_m^{(p)} = \frac{\pi}{2\mathrm{i}}\left[\frac{v_{\perp}}{c}J_{m+p}(\lambda_1\rho_0) - \frac{v_{\|}\lambda_1}{ck_z}J_m(\lambda_1\rho_0)\right], \qquad \varepsilon_0 = \varepsilon_1. \tag{18}$$

The Fourier coefficients for the fields determined by relations (15), (17) have poles corresponding to the zeros of the function α_m appearing in the denominators of (10) and (14). It can be seen that this function has zeros only under the conditions $\lambda_1^2 < 0 < \lambda_0^2$. In particular, from here, as a necessary condition, we should have $\varepsilon_1 < \varepsilon_0$. For the corresponding modes the Fourier coefficients of the fields depend on the radial coordinate through the MacDonald function $K_{\nu}(|\lambda_1|\rho)$ with $\nu = m, m \pm 1$, and they are exponentially damped with the distance from the cylinder surface. These modes are the eigenmodes of the dielectric cylinder and they correspond to the waves propagating inside the cylinder. Below, in the consideration of the intensity for the radiation to the exterior medium, we will neglect the contribution of the poles corresponding to these modes.

3. Radiation intensity

Having the electromagnetic fields we can investigate the intensity of the radiation propagating in the exterior medium. As we have mentioned before, for $\lambda_1^2 < 0$ the corresponding Fourier components are exponentially damped for large values ρ, and the radiation is present only under the condition $\lambda_1^2 > 0$. The average energy flux per unit time through the cylindrical surface of radius ρ, coaxial with the dielectric cylinder, is given by the Poynting vector

$$I = \frac{c}{2T}\int_0^T \mathrm{d}t \int_{-\infty}^{\infty} [\mathbf{E}\times\mathbf{H}]\cdot\mathbf{n}_{\rho}\rho\,\mathrm{d}z, \tag{19}$$

being $T = 2\pi/\omega_0$ the period for the transverse motion of the charge. Substituting the corresponding Fourier expansions of the fields and using the asymptotic expressions of the Hankel functions, at large distances from the cylinder for the radiation intensity we find

$$I = \frac{q^2c^2}{\pi^2}\sum_{m=0}^{\infty}{}' \int_{\lambda_1^2>0}\frac{\mathrm{d}k_z}{\varepsilon_1|\omega_m(k_z)|}\left[\frac{\omega_m^2(k_z)}{c^2}\varepsilon_1\left|D_m^{(+1)} - D_m^{(-1)}\right|^2 + k_z^2\left|D_m^{(+1)} + D_m^{(-1)}\right|^2\right], \tag{20}$$

where the prime over the sum means that the term with $m = 0$ should be taken with the weight $1/2$ and the coefficients $D_m^{(p)}$ are defined by formulae (16).

First let us consider the special case $\omega_0 = 0$ for a fixed ρ_0, which corresponds to a charge moving with constant velocity $v_{\|}$ on a straight line $\rho = \rho_0$ parallel to the cylinder axis. For this case, from (6), $\omega_m(k_z) = k_z v_{\|}$ and expressions (8) take the form $\lambda_j^2 = k_z^2\left(\beta_{j\|}^2 - 1\right)$ with the notation

$$\beta_{j\|} = \frac{v_{\|}}{c}\sqrt{\varepsilon_j}, \qquad j = 0, 1. \tag{21}$$

From the condition $\lambda_1^2 > 0$ it follows that the radiation in the exterior medium is present only under the Cherenkov condition for the particle velocity, $v_{\|}$, and dielectric permittivity ε_1 for the surrounding medium: $\beta_{1\|} > 1$. Introducing the angle ϑ of the wave vector with the cylinder axis, from the relation $k_z = \omega/v_{\|}$ it follows that $\cos\vartheta = \beta_{1\|}^{-1}$, and the radiation propagates along the Cherenkov cone of the external medium. Since for the case under consideration one has $v_\perp = 0$, the first term on the right of formula (14) vanishes and the expression for the coefficients $D_m^{(p)}$ is written in the form

$$D_m^{(p)} = -\frac{v_\parallel \pi}{2\mathrm{i}c}\sqrt{\beta_{1\parallel}^2 - 1}J_m\big(\lambda_1^{(0)}\rho_0\big) + v_\parallel \frac{H_m\big(\lambda_1^{(0)}\rho_0\big)}{cV_m^H} \times \left[pk_z \frac{J_m\big(\lambda_0^{(0)}\rho_1\big)J_{m+p}\big(\lambda_0^{(0)}\rho_1\big)}{\rho_1\alpha_m V_{m+p}^H} + \frac{\pi}{2\mathrm{i}}V_m^J\sqrt{\beta_{1\parallel}^2 - 1}\right], \quad (22)$$

where

$$\lambda_1^{(0)} = k_z\sqrt{\beta_{1\parallel}^2 - 1}, \qquad \lambda_0^{(0)} = \begin{cases} k_z\sqrt{\beta_{0\parallel}^2 - 1}, & \beta_{0\parallel} > 1, \\ i|k_z|\sqrt{1-\beta_{0\parallel}^2}, & \beta_{0\parallel} < 1. \end{cases} \quad (23)$$

The substitutions $\lambda_j \to \lambda_j^{(0)}$ should also be made in the formulae for V_m^J, V_{m+p}^H and α_m. Changing the integration variable, from (20) for the corresponding radiation intensity we find

$$I = \frac{2q^2}{\pi^2 v_\parallel}\sum_{m=0}^{\infty}{}' \int_{\beta_{1\parallel}>1} \mathrm{d}\omega\, \omega\left[\left|D_m^{(+1)} - D_m^{(-1)}\right|^2 + \beta_{1\parallel}^{-2}\left|D_m^{(+1)} + D_m^{(-1)}\right|^2\right], \quad (24)$$

where in formula (22) for the coefficients $D_m^{(p)}$ the expressions (23) should be used with $k_z = \omega/v_\parallel$. In the limit $\rho_1 \to 0$ the only contribution to the radiation intensity comes from the first term on the right-hand side of formula (22) and, after the summation over m by using the formula $\sum_{m=-\infty}^{+\infty} J_m^2(x) = 1$, we obtain the standard expression for the intensity of the Cherenkov radiation in a homogeneous medium. For small values of the cylinder radius, the part in the radiation intensity induced by the presence of the cylinder vanishes as ρ_1^{2m} for the harmonics $m \geqslant 1$ and as ρ_1^2 for $m = 0$.

Now we return to the general case of the particle motion along the helix. First we consider the contribution of the mode with $m = 0$ to the radiation intensity given by (20). For this mode one has $\omega_m(k_z) = k_z v_\parallel$ and, as in the previous case, from the condition $\lambda_1^2 > 0$ it follows that in the exterior region, the corresponding radiation is present only under the condition $\beta_{1\parallel} > 1$. This radiation propagates along the Cherenkov cone $\vartheta = \vartheta_0 \equiv \arccos\big(\beta_{1\parallel}^{-1}\big)$ of the external medium. By using the expressions for the coefficients $D_m^{(p)}$ and introducing a new integration variable $\omega = |k_z|v_\parallel$, for the radiation intensity at $m = 0$ one obtains

$$I_{m=0} = \frac{q^2}{v_\parallel}\int_{\beta_{1\parallel}>1} \frac{\omega\, \mathrm{d}\omega}{\varepsilon_1}\left[\beta_{1\perp}^2|U_1(\omega)|^2 + \left|U_2(\omega) + \mathrm{i}\sqrt{\beta_{1\parallel}^2 - 1}U_0(\omega)\right|^2\right], \quad (25)$$

with the notations

$$U_l(\omega) = J_l\big(\lambda_1^{(0)}\rho_0\big) - H_l\big(\lambda_1^{(0)}\rho_0\big)\frac{V_l^J}{V_l^H}, \qquad l = 0, 1,$$
$$U_2(\omega) = \frac{2(\varepsilon_0 - \varepsilon_1)\omega J_0\big(\lambda_0^{(0)}\rho_1\big)J_1\big(\lambda_0^{(0)}\rho_1\big)H_0\big(\lambda_1^{(0)}\rho_0\big)\big(\pi\rho_1 V_0^H\big)^{-1}}{\varepsilon_1\lambda_0^{(0)}J_0\big(\lambda_0^{(0)}\rho_1\big)H_1\big(\lambda_1^{(0)}\rho_1\big) - \varepsilon_0\lambda_1 J_1\big(\lambda_0^{(0)}\rho_1\big)H_0\big(\lambda_1^{(0)}\rho_1\big)}. \quad (26)$$

In formulae (26), $\lambda_j^{(0)}$ are defined by relations (23) with $k_z = \omega/v_\parallel$ and

$$\beta_{j\perp} = \frac{v_\perp}{c}\sqrt{\varepsilon_j}. \quad (27)$$

In the limit $\rho_1 \to 0$, by using the formulae for the Bessel functions for small values of the argument, from formula (25) we find

$$I_{m=0} \approx I_0^{(0)} + \frac{\pi q^2\rho_1^2}{2v_\parallel}\int_{\beta_{1\parallel}>1} \mathrm{d}\omega \frac{\varepsilon_1 - \varepsilon_0}{\varepsilon_1^2}\omega^3\left(\beta_{1\parallel}^2 - 1\right)^2 J_0(\lambda_1\rho_0)Y_0(\lambda_1\rho_0), \quad (28)$$

where $Y_0(x)$ is the Neumann function, and

$$I_0^{(0)} = \frac{q^2}{v_\parallel} \int_{\beta_{1\parallel}>1} \mathrm{d}\omega \frac{\omega}{\varepsilon_1} \left[\beta_{1\perp}^2 J_1^2\left(\lambda_1^{(0)}\rho_0\right) + \left(\beta_{1\parallel}^2 - 1\right) J_0^2\left(\lambda_1^{(0)}\rho_0\right)\right], \tag{29}$$

is the radiation intensity at $m = 0$ in the case of the charge motion in a homogeneous medium with $\varepsilon_0 = \varepsilon_1$.

For the radiation at $m \neq 0$ harmonics, from the condition $\lambda_1^2 > 0$ one obtains the quadratic inequality with respect to k_z:

$$k_z^2\left(1 - \beta_{1\parallel}^{-2}\right) + 2k_z m\omega_0/v_\parallel + (m\omega_0/v_\parallel)^2 > 0. \tag{30}$$

It is convenient to write the solution to this inequality in terms of a new variable ϑ, $0 \leqslant \vartheta \leqslant \pi$, defined in accordance with the relation

$$k_z = \frac{m\omega_0}{c} \frac{\sqrt{\varepsilon_1}\cos\vartheta}{1 - \beta_{1\parallel}\cos\vartheta}. \tag{31}$$

Now the function $\omega_m(k_z)$ is presented in the form

$$\omega_m(k_z) = \frac{m\omega_0}{1 - \beta_{1\parallel}\cos\vartheta}, \tag{32}$$

and the quantities k_z and $\omega_m(k_z)$ are connected by the relation $k_z = \omega_m(k_z)\sqrt{\varepsilon_1}\cos\vartheta/c$. In terms of the new variable ϑ, at large distances from the charge trajectory the dependence of elementary waves on the spacetime coordinates has the form

$$\exp[\omega_m(k_z)\sqrt{\varepsilon_1}(\rho\sin\vartheta + z\cos\vartheta - ct/\sqrt{\varepsilon_1})/c], \tag{33}$$

which describes wave with the frequency

$$\omega_m = |\omega_m(k_z)| = \frac{m\omega_0}{|1 - \beta_{1\parallel}\cos\vartheta|}, \qquad m = 1, 2, \ldots, \tag{34}$$

propagating at the angle ϑ to the z-axis. Formula (34) describes the normal Doppler effect in the cases $\beta_{1\parallel} < 1$ and $\beta_{1\parallel} > 1$, $\vartheta > \vartheta_0$, and the anomalous Doppler effect inside the Cherenkov cone, $\vartheta < \vartheta_0$, in the case $\beta_{1\parallel} > 1$.

Changing the integration over k_z in (20) to the integration over ϑ in accordance with (31), the radiation intensity at $m \neq 0$ harmonics is presented in the form

$$I_{m\neq 0} = \sum_{m=1}^{\infty} \int \mathrm{d}\Omega \frac{\mathrm{d}I_m}{\mathrm{d}\Omega}, \tag{35}$$

where $\mathrm{d}\Omega = \sin\vartheta\,\mathrm{d}\vartheta\,\mathrm{d}\phi$ is the solid angle element, and

$$\frac{\mathrm{d}I_m}{\mathrm{d}\Omega} = \frac{q^2\omega_0^2 m^2\sqrt{\varepsilon_1}}{2\pi^3 c|1 - \beta_{1\parallel}\cos\vartheta|^3}\left[\left|D_m^{(1)} - D_m^{(-1)}\right|^2 + \left|D_m^{(1)} + D_m^{(-1)}\right|^2\cos^2\vartheta\right], \tag{36}$$

is the average power radiated by the charge at a given harmonic m into a unit solid angle. In formulae (16) for the coefficients $D_m^{(\pm 1)}$, the quantities λ_0 and λ_1 are expressed in terms of ϑ as

$$\lambda_0 = \frac{m\omega_0}{c} \frac{\sqrt{\varepsilon_0 - \varepsilon_1\cos^2\vartheta}}{1 - \beta_{1\parallel}\cos\vartheta}, \qquad \lambda_1 = \frac{m\omega_0}{c} \frac{\sqrt{\varepsilon_1}\sin\vartheta}{1 - \beta_{1\parallel}\cos\vartheta}. \tag{37}$$

Hence, under the Cherenkov condition, $\beta_{1\parallel} > 1$, the total radiation intensity is presented in the form

$$I = I_0 + I_{m\neq 0}, \tag{38}$$

where the first term on the right-hand side is given by formula (25) and describes the radiation with a continuous spectrum propagating along the Cherenkov cone $\vartheta = \vartheta_0$ of the external medium. The second term describes the radiation, which, for a given angle ϑ, has a discrete spectrum determined by formula (34). If the Cherenkov condition is not satisfied ($\beta_{1\parallel} < 1$) the first term is absent and only the modes with $m \neq 0$ contribute to the radiation intensity.

For a charge moving in a homogeneous medium with dielectric permittivity ε_1, one has $\varepsilon_0 = \varepsilon_1$, and using formula (18) for the coefficients $D_m^{(p)}$, from formula (36) we obtain

$$\frac{\mathrm{d}I_m^{(0)}}{\mathrm{d}\Omega} = \frac{q^2\omega_0^2 m^2}{2\pi c\sqrt{\varepsilon_1}|1-\beta_{1\parallel}\cos\vartheta|^3}\left[\beta_{1\perp}^2 J_m'^2(\lambda_1\rho_0) + \left(\frac{\cos\vartheta-\beta_{1\parallel}}{\sin\vartheta}\right)^2 J_m^2(\lambda_1\rho_0)\right]. \tag{39}$$

In the case $\varepsilon_1 = 1$ this formula is derived in [21] (see also [3–5]). For a charge with a purely transversal motion ($v_\parallel = 0$) one has $D_m^{(p)} = (v_\perp/c)B_m^{(p)}$, and from (36) as a special case we obtain the formula derived in [14].

4. Features of the radiation

In this section, on the basis of the formulae given before, we discuss the characteristic features of the radiation intensity. For a non-relativistic motion, $\beta_{1\perp}, \beta_{1\parallel} \ll 1$, from the general formula (36) we find

$$\frac{\mathrm{d}I_m}{\mathrm{d}\Omega} \approx \frac{2q^2c(m\beta_{1\perp}/2)^{2(m+1)}}{\pi\rho_0^2\varepsilon_1^{3/2}(m!)^2}\left[1+\frac{\varepsilon_1-\varepsilon_0}{\varepsilon_0+\varepsilon_1}\left(\frac{\rho_1}{\rho_0}\right)^{2m}\right]^2(1+\cos^2\vartheta)\sin^{2(m-1)}\vartheta, \tag{40}$$

and the contribution of the harmonics with $m > 1$ is small compared to that in the fundamental one, $m = 1$. Note that the part of the radiation intensity with the first term in square brackets of (40) corresponds to the case of the motion in homogeneous medium with permittivity ε_1, and the part with the second term is induced by the presence of the cylinder with permittivity ε_0.

In the limit $\rho_1 \to 0$, in the general formula (36) we use the asymptotic expressions for the Bessel functions for small values of the argument. After long calculations it can be seen that the difference between the radiation intensities in the cases when the cylinder is present and absent, $\mathrm{d}I_m/\mathrm{d}\Omega - \mathrm{d}I_m^{(0)}\big/\mathrm{d}\Omega$, vanishes as ρ_1^{2m} for $m \geqslant 1$. In the same limit and for the radiation corresponding to $m = 0$, the part induced by the cylinder is given in formula (28) and vanishes like ρ_1^2.

Now let us consider the behavior of the radiation intensity near the Cherenkov angle when $|1-\beta_{1\parallel}\cos\vartheta| \ll 1$. Using the asymptotic formulae for the cylinder functions for large values of the argument it can be seen that in this limit

$$\frac{\mathrm{d}I_m}{\mathrm{d}\Omega} \propto |1-\beta_{1\parallel}\cos\vartheta|^{-2}. \tag{41}$$

Note that for the radiation intensity in a homogeneous medium with dielectric permittivity ε_1 we have the same behavior. In accordance with (34), near the Cherenkov cone the frequencies of the radiated photons are large and the dispersion of the dielectric permittivity ε_1 should be taken into account. The corresponding angles are determined implicitly from the condition $\omega_m \gtrsim \omega_d$, by using formula (34) and frequency dependence of the permittivity $\varepsilon_1 = \varepsilon_1(\omega_m)$. Here ω_d is the characteristic frequency of the dispersion. Note that for the charge helical motion inside the dielectric cylinder ($\rho_0 < \rho_1$) the behavior of the radiation intensity near the Cherenkov cone is radically different for the cases $\beta_{0\parallel} > 1$ and $\beta_{0\parallel} < 1$ (see [15]). In the first case the intensity behaves like $|1-\beta_{1\parallel}\cos\vartheta|^{-4}$, whereas in the second case the intensity behaves as $|1-\beta_{1\parallel}\cos\vartheta|^{-4}\exp\left[-2(\omega_m/v_\parallel)(\rho_1-\rho_0)\sqrt{1-\beta_{0\parallel}^2}\right]$, with ω_m given by (34).

By using Debye's asymptotic expansions for the Bessel and Neumann functions, in [15], in the case of helical motion inside a dielectric cylinder, it has been shown that under the condition $|\lambda_1|\rho_1 < m$, at points where the real part of the function α_m, given by formula (11), is equal to zero, the contribution of the imaginary part of this function into the coefficients $D_m^{(p)}$ can be exponentially large for large values m. This leads to the appearance of strong narrow peaks in the angular distribution for the radiation intensity at a given harmonic m. The condition for the real part of the function α_m to be zero has the form

$$\sum_{l=\pm 1}\left[\frac{\lambda_1}{\lambda_0}\frac{J_{m+l}(\lambda_0\rho_1)Y_m(\lambda_1\rho_1)}{J_m(\lambda_0\rho_1)Y_{m+l}(\lambda_1\rho_1)} - 1\right]^{-1} = \frac{2\varepsilon_0}{\varepsilon_1 - \varepsilon_0}, \tag{42}$$

where $Y_\nu(x)$ is the Neumann function. This equation is obtained from the equation determining the eigenmodes for the dielectric cylinder by the replacement $H_m \to Y_m$. Equation (42) has no solutions for $\lambda_0^2 < 0$, which is possible only for $\varepsilon_0 < \varepsilon_1$. Hence, the above-mentioned possibility for the appearance of peaks is not realized for the case $\lambda_0^2 < 0$ which, in accordance with (37), corresponds to the angular region $\cos^2\vartheta > \varepsilon_0/\varepsilon_1$.

Under the condition (42) for the coefficient α_m one has

$$\alpha_m \approx \frac{\mathrm{i}\lambda_0 J_m(\lambda_0\rho_1)}{\pi\rho_1}\sum_{l=\pm 1} l\frac{J_{m+l}(\lambda_0\rho_1)}{\left(V_{m+l}^Y\right)^2}, \tag{43}$$

where V_m^Y is defined by formula (9) with $F = Y$. For large values m, from (43) we have the estimate $\alpha_m \propto \exp[-2m\zeta(\lambda_1\rho_1/m)]$, with

$$\zeta(z) = \ln\frac{1+\sqrt{1-z^2}}{z} - \sqrt{1-z^2}, \tag{44}$$

and this coefficient is exponentially small. Now, by using the asymptotic formulae for the Bessel functions, we can see that under the conditions

$$|\lambda_1|\rho_0 < m < \lambda_0\rho_1, \tag{45}$$

for the angles being the solutions of equation (42), one has $D_m^{(p)} \propto \exp[m\zeta(\lambda_1\rho_0/m)]$ and, hence, for the radiation intensity

$$\frac{\mathrm{d}I_m}{\mathrm{d}\Omega} \propto \exp[2m\zeta(\lambda_1\rho_0/m)]. \tag{46}$$

Note the when the charge moves inside the cylinder ($\rho_0 < \rho_1$), for the appearance of the peaks we have two possibilities [15]. In the first case, corresponding to $|\lambda_1|\rho_1 < m < |\lambda_0|\rho_0$, at the peaks the radiation intensity behaves as $\exp[2m\zeta(\lambda_1\rho_1/m)]$. By taking into account that the function $\zeta(z)$ is monotonically decreasing and comparing this estimate with (46), we conclude that in this case the peaks in the radiation intensity for the motion inside the cylinder are stronger. The second case corresponds to $|\lambda_1|\rho_1, |\lambda_0|\rho_0 < m < |\lambda_0|\rho_1$, and the radiation intensity at the peaks behaves like $\exp\{2m[\zeta(\lambda_1\rho_1/m) - \zeta(\lambda_0\rho_0/m)]\}$.

From (45), by taking into account formulae (37), as necessary conditions for the presence of the strong narrow peaks in the angular distribution for the radiation intensity one has

$$\frac{\omega_0\rho_0}{c}\sqrt{\varepsilon_1}\sin\vartheta < |1-\beta_{1\|}\cos\vartheta| < \frac{\omega_0\rho_1}{c}\sqrt{\varepsilon_0 - \varepsilon_1\cos^2\vartheta}. \tag{47}$$

These conditions can be satisfied only if we have

$$\varepsilon_0 > \varepsilon_1, \qquad \tilde{v}\sqrt{\varepsilon_0}/c > 1, \tag{48}$$

where $\tilde{v} = \sqrt{v_\|^2 + \omega_0^2\rho_1^2}$ is the velocity of the charge image on the cylinder surface. The second condition in (48) is the Cherenkov condition for the velocity of the charge image on the cylinder surface and dielectric permittivity of the cylinder. From (47), it follows that when the Cherenkov condition for the velocity of the charge and dielectric permittivity of the surrounding medium is not satisfied, $v\sqrt{\varepsilon_1}/c < 1$, the possible strong peaks are located in the angular region defined by the inequality

$$\left|\frac{\tilde{v}}{c}\sqrt{\varepsilon_1}\cos\vartheta - \frac{v_\parallel}{\tilde{v}}\right| < \frac{\tilde{v}_\perp}{\tilde{v}}\sqrt{\frac{\tilde{v}^2}{c^2}\varepsilon_0 - 1}, \tag{49}$$

with $\tilde{v}_\perp = \omega_0\rho_1$. If the Cherenkov condition $v\sqrt{\varepsilon_1}/c > 1$ is satisfied, in addition to this inequality we also need to have the condition

$$\left|\frac{v}{c}\sqrt{\varepsilon_1}\cos\vartheta - \frac{v_\parallel}{v}\right| > \frac{v_\perp}{v}\sqrt{\frac{v^2}{c^2}\varepsilon_1 - 1}. \tag{50}$$

In order to estimate the angular widths for the peaks, we note that near them the main contribution to the radiation intensity comes from the terms in the coefficients $D_m^{(\pm)}$ containing in the denominators the function α_m. Expanding this function near the angle corresponding to the peak, $\vartheta = \vartheta_p$, we see that the angular dependence of the radiation intensity near the peak has the form

$$\frac{\mathrm{d}I_m}{\mathrm{d}\Omega} \propto \frac{1}{(\vartheta - \vartheta_p)^2/b_p^2 + 1}\left(\frac{\mathrm{d}I_m}{\mathrm{d}\Omega}\right)_{\vartheta=\vartheta_p}, \tag{51}$$

where $b_p \propto \exp[-2m\zeta(\lambda_1\rho_1/m)]$. Hence, the angular widths of the peaks are of the order $\Delta\vartheta \propto \exp[-2m\zeta(\lambda_1\rho_1/m)]$. From estimate (46) it follows that at the peaks the angular density of the radiation intensity exponentially increases with increasing m. However, one has to take into account that in realistic situations the growth of the radiation intensity is limited by several factors. In particular, the factor which limits the increase, is the imaginary part for the dielectric permittivity ε_j'', $j = 0, 1$. This leads to additional terms in the denominator of formula (51), proportional to the ratio $\varepsilon_j''/\varepsilon_j'$, where ε_j' is the real part of the dielectric permittivity. As a result for $b_p^2 < \varepsilon_j''/\varepsilon_j'$ the intensity and the width of the peak is determined by these terms and the saturation of the radiation intensity at the peak takes place.

By using the general formula (36), we have carried out numerical calculations of the radiation intensity at a given harmonic m for various values of the parameters. As an example, in figure 1, we have plotted the dependence of the angular density for the number of the radiated quanta

$$\frac{\mathrm{d}N_m}{\mathrm{d}\Omega} = \frac{1}{\hbar\omega_m}\frac{\mathrm{d}I_m}{\mathrm{d}\Omega}, \tag{52}$$

as a function of the angle $0 \leqslant \vartheta \leqslant \pi$ for $\beta_{1\perp} = 0.9$, $\rho_1/\rho_0 = 0.95$, $m = 10$. The full and dashed curves correspond to the cases $\varepsilon_0/\varepsilon_1 = 3$ and $\varepsilon_0/\varepsilon_1 = 1$ (homogeneous medium), respectively. The left panel is plotted for the longitudinal component $\beta_{1\parallel} = 0.5$. In order to compare with the radiation intensity in the case of purely transversal motion, we have plotted on the right panel the corresponding graphs for $\beta_{1\parallel} = 0$. In this case the curves are symmetric with respect to the rotation plane $\vartheta = \pi/2$. In both cases strong narrow peaks appear when the dielectric cylinder is present. At the peaks of the right panel one has $\vartheta \approx 0.5435$, $(T\sqrt{\varepsilon_1}\hbar c/q^2)\,\mathrm{d}N_m/\mathrm{d}\Omega \approx 138$, with the width of the peak $\Delta\vartheta \approx 10^{-4}$, and $\vartheta \approx 0.973$, $(T\sqrt{\varepsilon_1}\hbar c/q^2)\,\mathrm{d}N_m/\mathrm{d}\Omega \approx 12.9$, $\Delta\vartheta \approx 6\cdot 10^{-3}$. For the left peak on the left panel one has $\vartheta \approx 0.695$, $(T\sqrt{\varepsilon_1}\hbar c/q^2)\,\mathrm{d}N_m/\mathrm{d}\Omega \approx 1.53$, $\Delta\vartheta \approx 0.05$; $\vartheta \approx 1.726$, $(T\sqrt{\varepsilon_1}\hbar c/q^2)\,\mathrm{d}N_m/\mathrm{d}\Omega \approx 1.67$, $\Delta\vartheta \approx 0.02$; $\vartheta \approx 1.842$, $(T\sqrt{\varepsilon_1}\hbar c/q^2)\,\mathrm{d}N_m/\mathrm{d}\Omega \approx 2.37$, $\Delta\vartheta \approx 0.01$. These numerical data are in good agreement with the analytic estimates given before. From presented graphs it is seen that, for angles away from the peaks the drift essentially amplifies the radiation intensity.

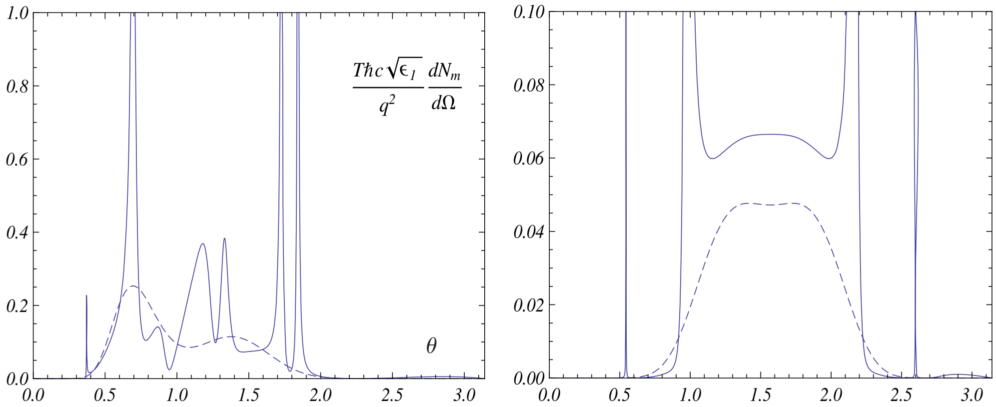

Figure 1. The dependence of the angular density for the number of radiated quanta, $(T\sqrt{\varepsilon_1}\hbar c/q^2)\,\mathrm{d}N_m/\mathrm{d}\Omega$, per period T of charge revolution as a function of the angle ϑ for $\beta_{1\perp} = 0.9, \rho_1/\rho_0 = 0.95, m = 10$. The left panel is plotted for $\beta_{1\|} = 0.5$ and the right panel is plotted for $\beta_{1\|} = 0$. Full and dashed curves correspond to the cases $\varepsilon_0/\varepsilon_1 = 3$ and $\varepsilon_0/\varepsilon_1 = 1$, respectively.

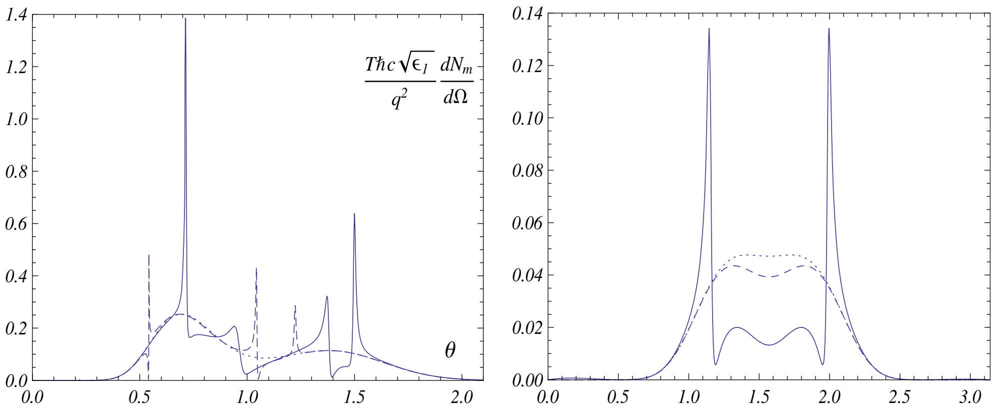

Figure 2. The dependence of the quantity $(T\sqrt{\varepsilon_1}\hbar c/q^2)\,\mathrm{d}N_m/\mathrm{d}\Omega$ on the angle ϑ for $\beta_{1\perp} = 0.9, m = 10, \varepsilon_0/\varepsilon_1 = 3$. For the left panel, $\beta_{1\|} = 0.5, \rho_1/\rho_0 = 0.8$ (full curve),$\rho_1/\rho_0 = 0.7$ (dashed curve). For the right panel, $\beta_{1\|} = 0, \rho_1/\rho_0 = 0.85$ (full curve), $\rho_1/\rho_0 = 0.8$ (dashed curve). On both panels the dotted lines present the corresponding quantities for the radiation in a homogeneous medium.

In order to illustrate the dependence of the radiation intensity on the ratio ρ_1/ρ_0, in figure 2, we have presented the angular dependence of the number of the radiated quanta for different values of this ratio. The graphs are plotted for $\beta_{1\perp} = 0.9, m = 10, \varepsilon_0/\varepsilon_1 = 3$. For the left panel $\beta_{1\|} = 0.5$. On this panel the full (dashed) curve corresponds to the value $\rho_1/\rho_0 = 0.8$ ($\rho_1/\rho_0 = 0.7$). For the right panel $\beta_{1\|} = 0$ and the full (dashed) curve corresponds to the value $\rho_1/\rho_0 = 0.85$ ($\rho_1/\rho_0 = 0.8$). On both panels the dotted lines present the corresponding quantities for the radiation in a homogeneous medium ($\varepsilon_0 = \varepsilon_1$). From these graphs we see that the influence of the dielectric cylinder is essential only in the case when the charge trajectory is sufficiently close to the cylinder surface.

5. Conclusion

The unique characteristics of synchrotron radiation, such as high intensity, high collimation and the wide spectral range, have resulted in its extensive applications. In the present paper, continuing our previous work on the influence of a medium on the parameters of the synchrotron radiation, we have investigated the properties of the radiation from a charged particle moving along a helical orbit around a dielectric cylinder. In order to evaluate the corresponding vector potential and the electromagnetic fields in the exterior medium, we have employed the Green function, previously investigated in [12]. These fields are presented in the form of the Fourier expansion (4) with the Fourier coefficients given by formulae (13), (15), (17). On the basis of these formulae, in section 3, we have investigated the spectral-angular distribution of the radiation propagating in the exterior medium. In the case when the Cherenkov condition for dielectric permittivity of the exterior medium and drift velocity of the charge is satisfied, $\beta_{1\|} > 1$, the radiation intensity is decomposed into two terms. The first one, given by formula (25), corresponds to the radiation with continuous spectrum propagating along the Cherenkov cone of the external medium. The second term in the expression for the total radiation intensity, given by formula (35), presents the contribution of the harmonics with $m \geqslant 1$. It describes the radiation, which for a given propagation direction characterized by the angle ϑ, has a discrete spectrum determined by formula (34). This formula describes the normal Doppler effect in the angular region outside the Cherenkov cone, $\vartheta > \vartheta_0$, and the anomalous Doppler effect in the region inside the cone. In the case $\beta_{1\|} < 1$, the first term in (38) is absent and the angular distribution of the radiation intensity at a given harmonic m is given by formula (36). In section 4 we have investigated characteristic features of the radiation. In the non-relativistic limit the general formula for the radiation intensity reduces to (40) and the main part of the radiation energy is emitted on the fundamental harmonic $m = 1$. For a fixed radius of the charge orbit and for small values of the cylinder radius, for a given harmonic the effects induced by the presence of the cylinder vanish as ρ_1^{2m}. As in the case of the particle motion inside the dielectric cylinder, under certain conditions on the parameters strong narrow peaks appear in the angular distribution of the radiation intensity. These peaks correspond to the values of the angle ϑ for which the real part of the function α_m from (11) vanishes. The corresponding condition (42) is obtained from the equation determining the eigenmodes for the dielectric cylinder replacing the Hankel functions by the Neumann ones. At the zeros of the real part of α_m the imaginary part of this function is exponentially small for large values m and this leads to strong angular peaks in the radiation intensity. By using asymptotic formulae for the cylindrical functions, we have specified the conditions for the appearance of the peaks and analytically estimated their heights and widths. In particular, we have shown that the peaks are present only when dielectric permittivity of the cylinder is greater than the permittivity for the surrounding medium and the Cherenkov condition is satisfied for the velocity of the charge image on the cylinder surface and the dielectric permittivity of the cylinder. These features are well confirmed by the results of the numerical calculations. These results show that the presence of the cylinder provides a possibility for an essential enhancement of the radiated power as compared to the radiation in a homogeneous medium.

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References

[1] Koch E E (ed) 1983 *Handbook on Synchrotron Radiation* (Amsterdam: North-Holland)
[2] Winick H and Doniach S (eds) 1980 *Synchrotron Radiation Research* (New York: Plenum)
[3] Sokolov A A and Ternov I M 1986 *Radiation from Relativistic Electrons* (New York: ATP)
[4] Ternov I M, Mikhailin V V and Khalilov V R 1985 *Synchrotron Radiation and Its Applications* (Amsterdam: Harwood Academic)
[5] Bordovitsyn V A (ed) 1999 *Synchrotron Radiation Theory and Its Development* (Singapore: World Scientific)
[6] Hofman A 2004 *The Physics of Synchrotron Radiation* (Cambridge: Cambridge University Press)
[7] Rullhusen P, Artru X and Dhez P 1998 *Novel Radiation Sources Using Relativistic Electrons* (Singapore: World Scientific)
[8] Tsytovich V N 1951 *Westnik MGU* **11** 27 (in Russian)
Kitao K 1960 *Prog. Theor. Phys.* **23** 759
Erber T, White D and Latal H G 1976 *Acta Phys. Austriaca* **45** 29
Schwinger J, Tsai W-Y and Erber T 1976 *Ann. Phys.* **96** 303
Erber T, White D, Tsai W-Y and Latal H G 1976 *Ann. Phys.* **102** 405
Rynne T M, Baumgartner G B and Erber T 1978 *J. Appl. Phys.* **49** 2233
[9] Mkrtchyan A R, Grigoryan L Sh, Saharian A A and Didenko A N 1991 *Acustica* **75** 1984
Saharian A A, Mkrtchyan A R, Gevorgian L A, Grigoryan L Sh and Khachatryan B V 2001 *Nucl. Instrum.Methods* B **173** 211
[10] Arzumanyan S R, Grigoryan L Sh and Saharian A A 1995 *Izv. Nats. Akad. Nauk Arm., Fiz.* **30** 99
Arzumanyan S R, Grigoryan L Sh and Saharian A A 1995 *J. Contemp. Phys.* **30** 99 (English Translation)
[11] Arzumanian S R, Grigoryan L Sh, Saharian A A and Kotanjyan Kh V 1995 *Izv. Nats. Akad. Nauk Arm., Fiz.* **30** 106
Arzumanian S R, Grigoryan L Sh, Saharian A A and Kotanjyan Kh V 1995 *J. Contemp. Phys.* **30** 106 (English Translation)
[12] Grigoryan L Sh, Kotanjyan A S and Saharian A A 1995 *Izv. Nats. Akad. Nauk Arm., Fiz.* **30** 239
Grigoryan L Sh, Kotanjyan A S and Saharian A A 1995 *J. Contemp. Phys.* **30** 239 (English Translation)
[13] Grigoryan L Sh, Khachatryan H F and Arzumanyan S R 1998 *Izv. Nats. Akad. Nauk Arm., Fiz.* **33** 267
Grigoryan L Sh, Khachatryan H F and Arzumanyan S R 1998 *J. Contemp. Phys.* **33** 267 (English Translation)
Grigoryan L Sh, Khachatryan H F and Arzumanyan S R 2002 *Izv. Nats. Akad. Nauk Arm., Fiz.* **37** 327
Grigoryan L Sh, Khachatryan H F and Arzumanyan S R 2002 *J. Contemp. Phys.* **37** 327 (English Translation)
Grigoryan L Sh, Khachatryan H F, Arzumanyan S R and Grigoryan M L 2006 *Nucl. Instrum.Methods* B **252** 50
Arzumanyan S R, Grigoryan L Sh, Khachatryan H F and Grigoryan M L 2008 *Nucl. Instrum. Methods* B **266** 3715
[14] Kotanjyan A S, Khachatryan H F, Petrosyan A V and Saharian A A 2000 *Izv. Nats. Akad. Nauk Arm., Fiz.* **35**
Kotanjyan A S, Khachatryan H F, Petrosyan A V and Saharian A A 2000 *J. Contemp. Phys.* **35** (English Translation)
Kotanjyan A S and Saharian A A 2001 *Izv. Nats. Akad. Nauk Arm., Fiz.* **36** 310
Kotanjyan A S and Saharian A A 2001 *J. Contemp. Phys.* **36** 310 (English Translation)
Kotanjyan A S and Saharian A A 2002 *Izv. Nats. Akad. Nauk Arm., Fiz.* **37** 135
Kotanjyan A S and Saharian A A 2002 *J. Contemp. Phys.* **37** 135 (English Translation)
Kotanjyan A S and Saharian A A 2002 *Mod. Phys. Lett.* A **17** 1323
Kotanjyan A S 2003 *Nucl. Instrum. Methods* B **201** 3
Saharian A A and Kotanjyan A S 2003 *Izv. Nats. Akad. Nauk Arm., Fiz.* **38** 288
Saharian A A and Kotanjyan A S 2003 *J. Contemp. Phys.* **38** 288 (English Translation)
Saharian A A and Kotanjyan A S 2004 *Nucl. Instrum. Methods* B **226** 351
[15] Saharian A A and Kotanjyan A S 2005 *J. Phys. A: Math. Gen.* **38** 4275
[16] Saharian A A, Kotanjyan A S and Grigoryan M L 2007 *J. Phys. A: Math. Theor.* **40** 1405
[17] Kotanjyan A S and Saharian A A 2007 *J. Phys. A: Math. Theor.* **40** 1405
[18] Arzumanyan S R, Grigoryan L Sh, Khachatryan H F, Kotanjyan A S and Saharian A A 2008 *Nucl. Instrum. Methods* B **266** 3703 (arXiv:0711.4673)
[19] Gevorgian L A and Pogosian P M 1994 *Izv. Nats. Akad. Nauk Arm., Fiz.* **19** 239
Gevorgian L A and Pogosian P M 1994 *J. Contemp. Phys.* **19** 239 (English Translation)
[20] Onuki H and Elleaume P (eds) 2003 *Undulators, Wigglers and Their Applications* (London: Taylor and Francis)
[21] Sokolov A A and Ternov I M 1968 *Z. Phys.* **211** 1